ALLOWANCE FOR THE INERTIA EFFECT IN THE PROBLEM OF THE DEFORMATION OF A VISCOELASTIC CYLINDER SUBJECTED TO CYCLIC LOADING

A. E. Segalov

UDC 532.135;534.1

The influence of the inertia effect on heating and creep acceleration in cylically loaded polymers is estimated; it is shown that for polymers whose compliance increases sharply on a certain temperature interval the temperature and resonance curves are multivalued.

The high viscosity and low thermal conductivity of rigid polymers leads to considerable heating when vibrational loads are applied. This considerably reduces their stiffness and facilitates deformation [1, 2]. Barenblatt [3] has investigated the effect of small secondary vibrational loads on the deformation of a polymer subjected to a slowly varying primary load on the frequency range on which the inertia effect may be neglected. As will be shown below, taking the inertia effect into account may lead to qualitatively new results.

On the assumption that the primary load is created by a heavy weight and the mass of the cylindrical polymer specimen is much less than the mass of the weight, it is possible to treat the massless cylinder and weight as a system with one degree of freedom [4, 5]. In this case the specimen is replaced by the equivalent (with respect to elastic and dissipative properties) spring.

A similar model was recently employed in [5], the change in the viscoelastic properties of the polymer specimen during deformation being taken into account simply by artificially introducing the dependence of the modulus of elasticity on strain amplitude.

Schapery [6] allowed for heating and its effect on the elastic and dissipative properties of the polymer in studying the steady-state vibrations of a massless slab with an attached concentrated mass. His computer calculations, in which he employed a specific form of the functions E'(T) and E"(T), showed that the temperature and strain amplitude may have a multivalued dependence on vibration frequency. Schapery's subsequent experiments [7] confirmed this result.

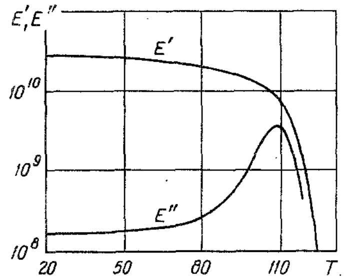

Fig. 1. Dynamic modulus E' (dyne/cm^2) and loss modulus E" (dyne/cm^2) as functions of temperature T (°C).

We have now shown with reference to the cyclic deformation of a polymer cylinder with an attached massive weight that if on a certain temperature interval the compliance of the polymer increases as a result of dissipative heating, the temperature and resonance curves will be multivalued.

1. A vibrational load $\sigma_0 S\cos\omega t$ is applied to a polymer cylinder with an attached massive weight. We consider the range of frequencies containing the first natural frequency of the weight–cylinder system. Because of the condition $M \gg \rho l S$ these frequencies are much less than the first natural frequency of the cylinder; accordingly, the deformation ε associated with the vibrational load is constant along the cylinder.

The vibration equation can be written in the form

$$m\frac{d^2\varepsilon}{dt^2}+\frac{E''}{\omega}\cdot\frac{d\varepsilon}{dt}+E'\varepsilon=\sigma_0\cos\omega t, \tag{1}$$

Institute of Mechanics, Moscow Lomonosov State University. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 21, No. 1, pp. 109-113, July, 1971. Original article submitted September 16, 1970.

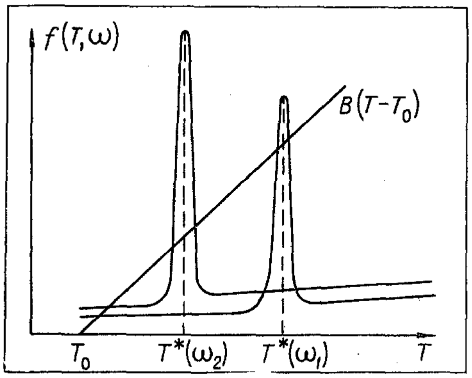

Fig. 2

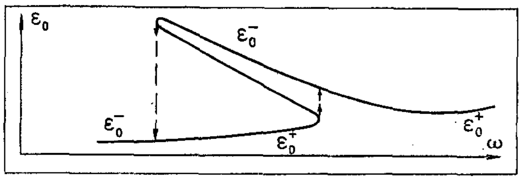

Fig. 3

Fig. 2. Heat release $f(T,\omega)$ and heat loss $B(T-T_0)$ functions versus temperature T (°C).

Fig. 3. Resonance curve.

where $m = lM/S$. In Fig. 1 [8] we have plotted the $E'(T)$ and $E''(T)$ dependences for polystyrene (we neglect the weak frequency dependence). This approximation is meaningful only for temperatures below the glass transition or softening points; accordingly, $E'(T) \gg E''(T)$.

Averaging the cylinder temperature T over a time interval large as compared with the vibration period, we obtain the energy balance equation

$$\rho c \frac{dT}{dt} = \frac{\omega E'' \varepsilon_0^2}{2A} - \frac{2\alpha}{r}(T - T_0). \tag{2}$$

Let the characteristic time of temperature variation, given by Eq. (2), be much greater than the time required to build up the forced vibrations. Then for the steady-state strain we obtain

$$\varepsilon = \frac{\sigma_0 \cos(\omega t - \delta)}{[(E' - m\omega^2)^2 + E''^2]^{1/2}} = \varepsilon_0 (T, \omega) \cos(\omega t - \delta). \tag{3}$$

Let the vibration frequency be less than the initial natural frequency of the weight–cylinder system ($\omega^2 < E'(T_0)/m$). As a result of the gradual heating of the cylinder and its loss of stiffness the natural frequency of the system falls. At the "resonance" temperature $T^*(\omega)$, at which $E'(T^*) = m\omega^2$, we get "parametric" resonance.

2. For determining the steady-state temperature $\theta(\omega)$ we have

$$B(\theta - T_0) = f(\theta, \omega),$$

where the parameter $B = 4\alpha A/r$ characterizes the heat transfer;

$$f(T, \omega) = \omega E''(T)\varepsilon_0^2 = \frac{\omega E'' \sigma^2}{(E' - m\omega^2)^2 + E''^2}.$$

A typical plot of the function f for two frequency values $\omega_2 > \omega_1$ is shown in Fig. 2. At a fixed vibration frequency either one or three ($\theta_1 < \theta_2 < \theta_3$) steady thermal states (in the degenerate case two) are possible. The steady state with temperature θ_2 is unstable, since in response to a slight fall in temperature heat transfer begins to predominate leading to cooling of the cylinder to the temperature θ_1 whereas a rise in temperature leads to the predominance of heat release and heating to θ_3. Thus, temporary overheating of the cyclically loaded element above the temperature θ_2 leads to a sharp rise in the steady-state operating temperature.

The multivaluedness of $\theta(\omega)$ leads to multivaluedness of the resonance curve $\varepsilon_0(\omega)$ (Fig. 3), but the section corresponding to the unstable $\theta_2(\omega)$ branch of the temperature curve cannot be realized. As the frequency increases, the resonance curve $\varepsilon_0^+(\omega)$ is followed, and as it decreases the other resonance curve $\varepsilon_0^-(\omega)$. Transition from one branch to the other may likewise result from a brief overheating or cooling of the cylinder.

Thus, in a cylinder subjected to cyclic loading at constant frequency the steady state depends on the initial temperature. Correspondingly, either a greater $\varepsilon_0^-(\omega)$ or a lesser $\varepsilon_0^+(\omega)$ strain amplitude may be established. If, however, the process results in the establishment of a steady-state temperature $\theta(\omega) > T^*(\omega)$, then the maximum strain amplitude will be reached during heating (at the "resonance" temperature

$T^*(\omega)$), with a subsequent decline to $\varepsilon_0(\omega)$. Thus, in connection with the design of cyclically loaded polymeric parts a certain intermediate state, reached during the transient process, may prove to be more dangerous than the final state corresponding to the maximum temperature.

3. Let us now consider the creep of a polymer specimen subjected to a considerable static stress σ^0 and a small secondary vibrational load ($\sigma^0 \gg \sigma_0$). The strain rate [9]

$$\frac{d\varepsilon^0}{dt} = F(\varepsilon^0)\exp\left(-\frac{U-\gamma\sigma^0}{RT}\right). \tag{4}$$

In this case the role of the vibrational load reduces merely to raising the temperature of the specimen and the creep rate depends essentially on which of the above steady-state thermal regimes – low temperature of high-temperature – is established.

In particular, the intense heat release and acceleration of creep observed in the later stages of the vibrocreep experiment described in [9] were evidently partially caused by the inertia effect. The original, purely viscous heating of the polymer reduced its modulus of elasticity and brought the system closer to parametric resonance, which then further accelerated the heating.

Thus, the inertia effect in cyclically loaded polymeric materials may lead to an additional increase in deformation.

NOTATION

T	is the cylinder temperature;
$E'(T)$	is the dynamic modulus;
$E''(T)$	is the loss modulus;
S, l, and r	are the cross-sectional area, length, and radius of the cylinder;
ρ and c	are the density and specific heat;
ω	is the frequency;
t	is the time;
M	is the mass of the weight;
σ_0 and ε_0	are the stress and strain amplitudes;
A	is the mechanical equivalent of heat;
α and T_0	are the heat transfer coefficient and ambient temperature;
B	is the heat transfer parameter;
$T^*(\omega)$	is the "resonance" temperature;
θ	is the steady-state cylinder temperature;
σ^0	is the static stress;
ε^0	is the creep strain;
U	is the activation energy;
R	is the universal gas constant;
γ	is a material constant.

LITERATURE CITED

1. E. A. W. Hoff, J. Appl. Chem., 2, No. 8 (1952).
2. J. F. Tormey and S. C. Britton, AIAA J., 1, No. 8 (1963).
3. G. I. Barenblatt, Prikl. Matem. i Mekhan., 30, No. 1 (1966).
4. S. P. Timoshenko and D. H. Young, Vibration Problems in Engineering, Van Nostrand, New York (1955).
5. V. A. Kuz'menko and V. A. Sitnyavskii, Prikl. Mekhan., 5, No. 2 (1969).
6. R. A. Schapery, Trans. ASME, Ser. E, J. Appl. Mech., 32, No. 3 (1965).
7. R. A. Schapery and D. E. Cantey, AIAA J., 4, No. 2 (1966).
8. M. Takaynagi, Mem. Eng. Kyushu Univ., 23, No. 1 (1963).
9. G. I. Barenblatt, Yu. S. Kozyrev, N. I. Malinin, D. Ya. Pavlov, and S. A. Shesterikov, Zh. Prikl. Mekhan. i Tekh. Fiz., No. 5 (1965).